

Improving Speed and Dexterity of Dynamic Motions in Bio-Inspired Robotic Limbs

by

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Submitted to the Department of Mechanical Engineering
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ABSTRACT

The translation of biological systems to robotic platforms requires deliberate design choices about which elements of the original system to replicate, omit, or modify. This thesis explores the refinement of that translation in the case of a bio-inspired robotic arm, focusing primarily on the addition of a 3 degree of freedom (DOF) wrist module using a streamlined co-design approach in which optimization results actively inform hardware design choices in the system. Central hardware elements designed include a custom motor module used for all DOFs in the wrist, and a modular wrist assembly that emphasizes range of motion, reconfigurability, and structural stability under high loads.

In conjunction with hardware design, new control strategies are developed to test how the wrist can be used most effectively; both as an active actuator and as a passive element that leverages momentum transfer, drawing inspiration from kinetic chain dynamics in human motion. System performance is tested and quantified in simulation as an addition to the MIT Biomimetic Robotics Lab's robotic table tennis system, in which the wrist (previously limited to a single degree of freedom) must precisely position and orient a paddle to return balls with a desired spin, velocity, and landing position during gameplay.

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Chapter 1

Introduction

As robotic systems become increasingly integrated with the human world, the design of robots that can both mimic and build upon human activity has been a focus in the field [1–3]. One key research interest of the MIT Biomimetic Robotics Lab, in which this thesis was conducted, is to design hardware exploring optimal methods of translation between biological and robotic systems, supported by deliberate design choices about which elements of the original system to replicate, omit, or modify [4]. While there is a great deal of value that can be found in diligent replication of biological analogues, many impressive robotic systems in this space feature meaningful departures; Boston Dynamics’ recent iterations of the Atlas humanoid platform, for example, have featured 360° mobility on most joints, enabling smooth and complex maneuvers beyond human ability [5].

The wrist plays a critical role in the accomplishment of both quasi-static and dynamic motions, from extreme athletic feats to everyday tasks, and its biological complexity makes it an especially interesting subject of bio-inspired design. The optimal translation to robotic systems of several phenomena that have been observed in patterns of human wrist usage is not yet clear, and offers an interesting area of exploration for engineers working on these systems.

1.1 Biomechanical basis

Biomechanical studies have found the wrist to be a central upstream player in hand and finger dexterity during fine and gross motor skill tests, with constraint of the wrist (through the presence of a splint, for example) noticeably impeding both the speed and precision with which tasks could be completed with the hand and fingers [6].

For repetitive, high-impact motions (such as the handling of a tennis racket during competitive gameplay), significant differences in wrist motion patterns can be identified between novice and expert athletes. Expert players typically perform backhand strokes with the wrist in extension (in which the back of the hand moves up, approaching the top of the forearm), and are actively increasing this degree of extension during the moment of impact. Contrastingly, novice players typically perform backhand strokes with the wrist in actively increasing flexion (in which the palm of the hand moves down, approaching the bottom of the forearm) during impact, and experience significantly higher rates of lateral

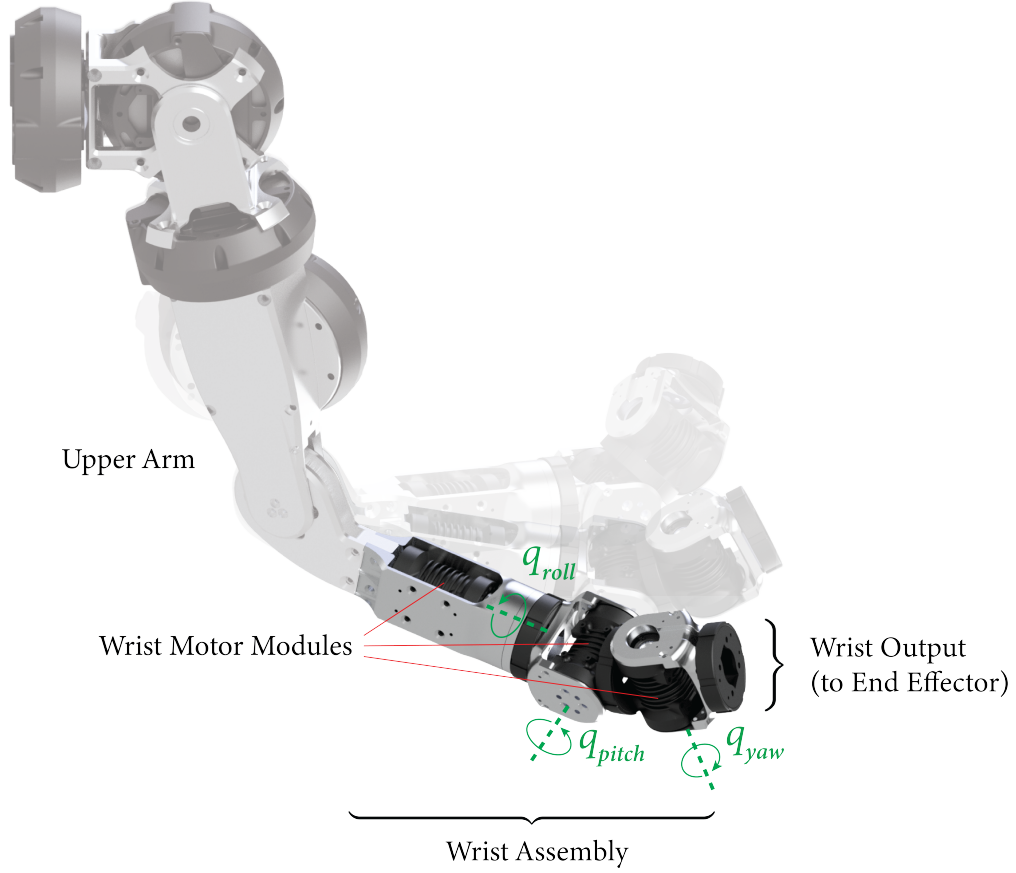


Figure 1.1: Overview of the wrist assembly described in this work. Note that the pitch and yaw degrees of freedom for this design are not necessarily determinate, since the position of the roll joint q_{roll} dictates which of the subsequent two joints governs each of these conventional rotations.

humeral epicondylitis, a painful overuse injury of the forearm and elbow [7]. This correlation suggests that wrist use patterns in humans may sometimes be adapted to injury prevention and pain reduction, both issues that do not concern robotic systems. The complexity of factors impacting human motions, such as in this case, serves as a reminder to the engineers of bio-inspired systems to remain considerate of the optimality, or lack thereof, of the pure replication of biological analogues.

General dynamic, high speed athletic motions in addition to striking, such as throwing, use the wrist to control momentum transfer and adjust hand position to improve accuracy at a late stage in a temporal muscle activation sequence often referred to as a kinetic chain [7,8].

In addition, performance properties of the wrist, such as stiffness, can be intentionally modulated via variable muscle activation in response to different loading conditions. Specifically, effective wrist stiffness can be increased in an effort to stabilize unstable loads [9].

1.2 Platform overview

This project aims to develop a modular hardware platform that can be used to thoroughly explore the role of the wrist in improving the capabilities of several robotic systems in the lab, and to offer preliminary results and analysis from some of these explorations. In particular, two sets of simulation experiments are conducted in this work: Firstly, the impact of wrist utilization on system performance for the lab’s high speed robotic table tennis system [10] is tested with the use of variable costs on wrist position deviation during trajectory optimization. Secondly, the utility of the wrist as a passive element is explored on the same system, using joint-specific multipliers on the torque limits of joints in the wrist.

An overview of the wrist hardware platform developed in this work is shown in Fig. 1.1. The wrist is mounted to the existing upper arm of the MIT Humanoid [3], with a 3 DOF shoulder and single DOF elbow. All upper arm joints are driven by U10 motor modules previously designed in the lab, capable of 34 Nm of peak torque and a 48 rad/s maximum speed [11]. With a total mass under 3 kg, this upper arm provides a strong foundation for the addition of a wrist assembly with similar torque density.

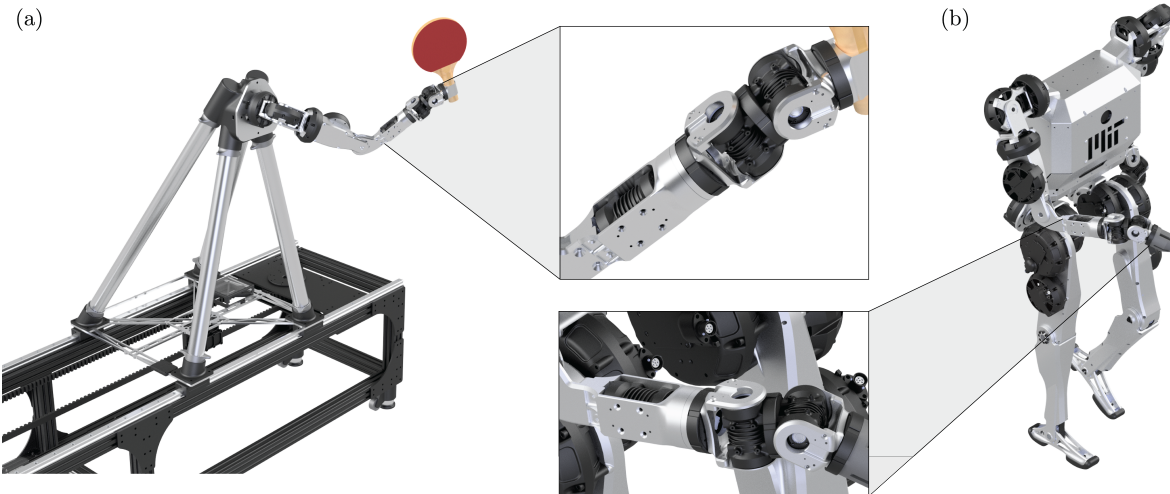


Figure 1.2: (a) One example deployment of this wrist module in the lab, as an addition to a high speed robotic table tennis system. This system previously featured a simpler 1 DOF wrist, allowing only roll of the paddle. With this upgraded wrist, simulation results have shown an expansion of the system’s workspace and an ability for last-minute adjustments to improve strike accuracy. Hardware experiments to validate these simulation results are an upcoming focus of this ongoing work. (b) An alternative deployment of this wrist module on the MIT Humanoid [3] platform, for use in mobile manipulation tasks. The current MIT Humanoid lacks wrist DOFs altogether, given its past focus on locomotion.

In the wrist, 3 degrees of freedom (DOFs) are joined in series, each driven by a custom motor module (discussed further in Chapter 2) with an inline gearbox for increased torque density. Each joint interfaces with the others through a low-profile, externally threaded torque transfer coupling (discussed further in Chapter 3), allowing joints in the wrist to be easily

added, removed, or reconfigured (for example, rotated in 90° increments). The wrist output interfaces with an additional torque-transfer coupling, enabling the use of interchangeable end effectors for a wide range of applications.

Chapter 2

Motor Module Design

To enable the desired performance of the wrist assembly while maintaining a small footprint, a new motor module with an inline gearbox was designed for this work. This design incorporates a prior planetary gearbox layout and a Mosrac U2523 motor into a single housing, extending the sun gear in the gearbox to interface with a rotary assembly through a critical shrink fit. In the early stages of design, prior to motor selection, a series of simulations with a simplified arm and wrist assembly was run with tunable motor performance limits. Simulation results informed the selection of a larger, higher torque motor out of two candidates. In subsequent stages, design specifics were further guided by principles of proper constraint [12], analytical and simulated models of performance, and the underlying goal of modularity, discussed in the previous chapter, to allow for easier translation between multiple robotic systems.

2.1 Prior work and additional needs

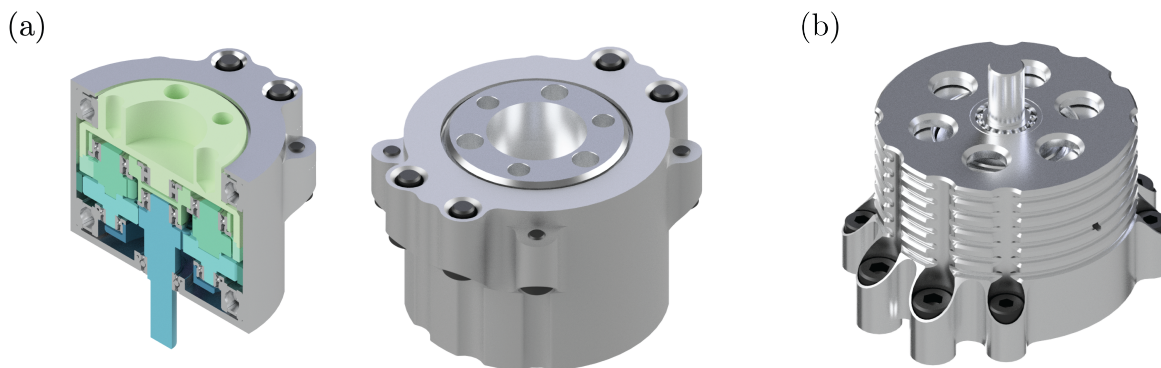


Figure 2.1: (a) Prior gearbox design by Nick Cerone used in this work. (b) Separate existing motor module used to drive gearbox pictured to the left in parallel through a spur gear transmission.

Existing work in the MIT Biomimetic Robotics Lab has yielded a wide range of custom actuators with impressive performance, driving systems like the MIT Humanoid and MIT

Mini Cheetah, as well as multiple manipulation platforms [13–15]. One existing actuator featuring a compound planetary gearbox [16] is of particular relevance to this project (shown in Fig. 2.1), given its small size and significant torque amplification. While the existing gearbox design is well suited to this application, the spur gear transmission (a feature resulting from the separation and parallel mounting of the motor and gearbox) is not geometrically compatible with a wrist. To maintain the torque density provided by this gearbox, a new module was designed for this project to mount both the gearbox and motor within one compact housing.

2.2 Preliminary simulations and motor selection

The size of motor used in this module was a critically important early decision, with a trade-off between sufficient torque output, sufficient speed, and minimal mass governing this selection. A set of potentially well-suited, readily accessible motors was narrowed down to two candidates described in Table 2.2, both frameless brushless DC motors in the U25 series by manufacturer Mosrac.

	Instant Max Torque	Geared Instant Max Torque	Max Speed	Weight
U2523	0.225 Nm	6.75 Nm	8000 rpm	0.06 kg
U2515	0.15 Nm	4.5 Nm	8000 rpm	0.036 kg

Table 2.1: Performance specifications for motor candidates.

Since both the U2515 and U2523 motors offer the same maximum speed, the remaining governing factor to analyze in this trade-off was the maximum torque output of each motor, and whether the smaller and lighter U2515 motor could supply the necessary torque to drive the system at its operating limits. To explore this performance with greater depth and precision than reasonably possible with analytical calculations by hand, preliminary trajectory optimizations were run with the U2515 as the actuator for each wrist joint.

The lab’s high speed robotic table tennis system (Fig. 1.2 (a)) was used in these optimizations, including a simplified representation of the wrist assembly mounted to an existing upper arm (including a 3 DOF shoulder and a single DOF elbow), all of which are mounted on the translating carriage of a large single DOF gantry.

Since a detailed description of simulation and trajectory optimization setup for other elements of this project is included in Chapter 4, this structure will be overviewed only briefly below. Trajectory optimizations were constructed using CasADi’s nonlinear optimization tools, and visualized in MuJoCo. For each trajectory, the system was constrained to achieve a certain desired state (position and velocity) at the time it would strike a table tennis ball, while adhering to the performance limits of each motor and the dynamics of the system. To examine the case of the U2515 motor, the torque limits of the wrist were set to 5 Nm, slightly above the maximum output of the U2515, with a hypothesis that insufficient performance given this limit would rule out the U2515 as a possibility. If performance at this limit proved sufficient, torque limits would be refined to locate an acceptable cutoff more precisely. Although not represented in these torque limits, significant unmodeled performance loss

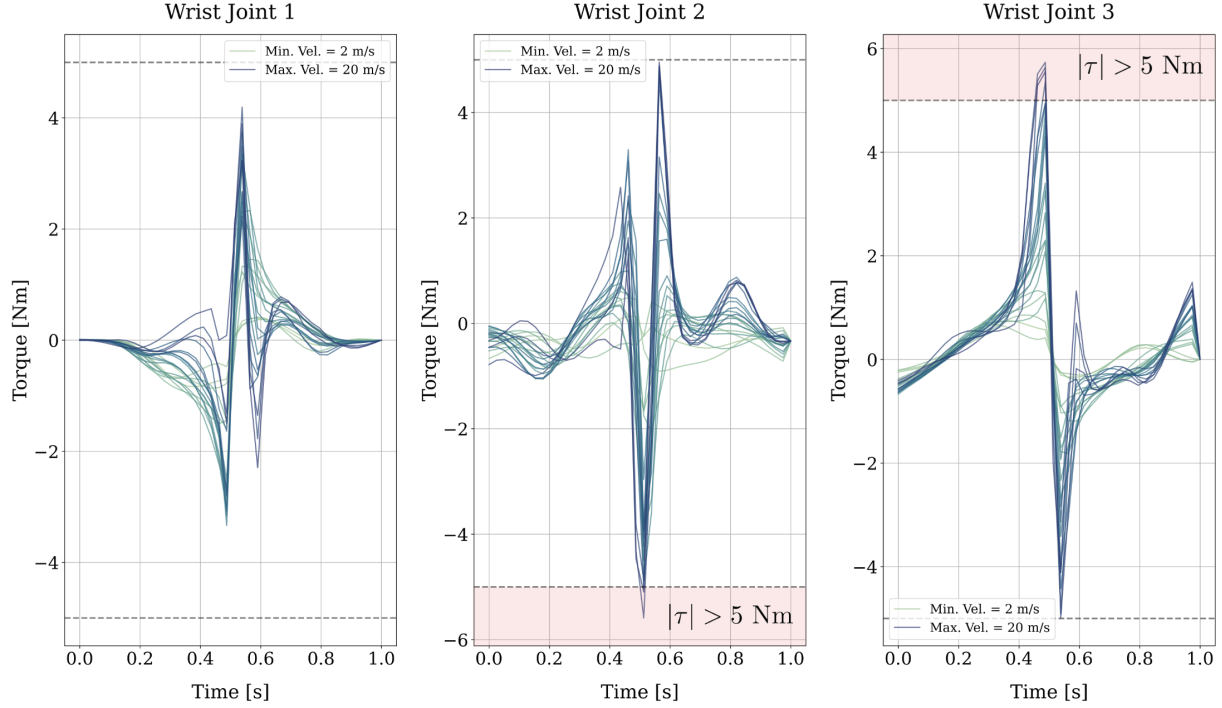


Figure 2.2: Torque command over time for a sweep of swings all reaching one representative target state at $t=0.5$ seconds. Target velocities range from 2 m/s to 20 m/s, indicated by the lightest and darkest curves, respectively.

resulting from the imperfect efficiency of the gearboxes in the wrist motor modules was also considered during all analysis.

A 2 dimensional sweep of trajectory optimizations was conducted, with target positions ranging throughout the workspace of the platform and target velocities ranging from 2 m/s to 20 m/s (chosen to align with the performance of professional human table tennis players). The torque command from the wrist for each optimal trajectory was analyzed, with one subset of trajectories at a constant target position shown in Fig. 2.2. The system was unable to achieve the most ambitious velocity targets (those approaching 20 m/s) without violating the imposed constraints on wrist motor torque in the case of the U2515 motors. This result informed the decision to choose the U2523 motors for this system, with the necessity of their higher maximum torque output justifying their larger mass and volume.

2.3 Motor module overview

Shown in Fig. 2.3 and Fig. 2.5 are views of the motor module designed for this work with internal components visible, while Fig. 2.4 shows the exterior of the motor module housing.

Components within the module are grouped by subassembly and assigned individual labels in Fig. 2.5, which will be referenced in the description to follow. The existing compound planetary gearbox (subassembly C) is contained in the upper half of the housing (subassembly A), and connected through the shaft of a new extended sun gear (B1) to a rotary adapter (B2), which is mounted in turn to the inner surface of the rotor (B4). The stator of the motor

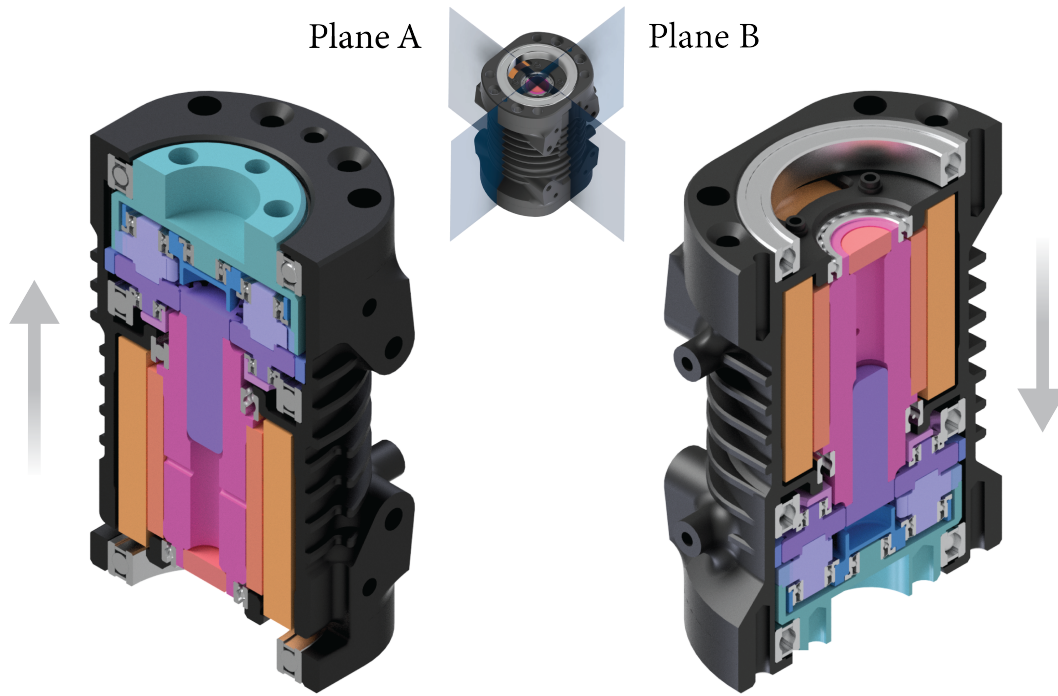


Figure 2.3: Cross sections of the motor module split along each of two perpendicular planes (labeled A and B above) collinear with the central axis of the motor. Note that the right and left cross sections have flipped vertical orientations relative to each other, in order to show a broader view of the module.

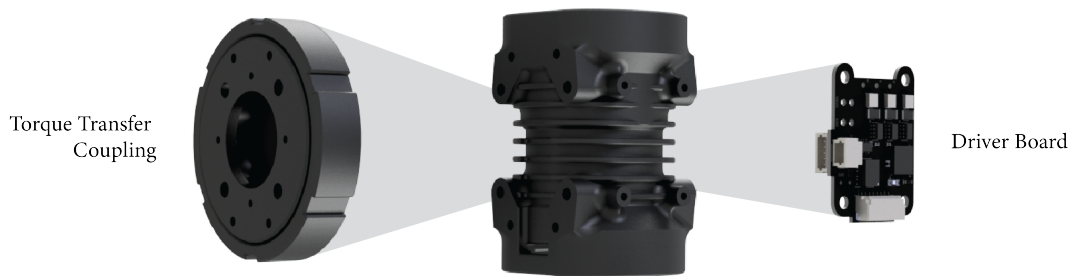


Figure 2.4: Externally mounted parts interfacing with the motor module, including a torque transfer coupling that allows modules to be mounted securely both to each other and to auxiliary wrist components, as well as the driver board for the module. Note that there are two symmetrical torque transfer coupling mounts on opposite sides of the motor module, though only one is visible from this view.

(B5) is contained in the lower half of the housing, and the encoder for the motor is mounted to the lower surface of the housing bottom (A3). A magnet for the encoder (B3) is mounted within the rotary shaft, along the axis of the sun gear.

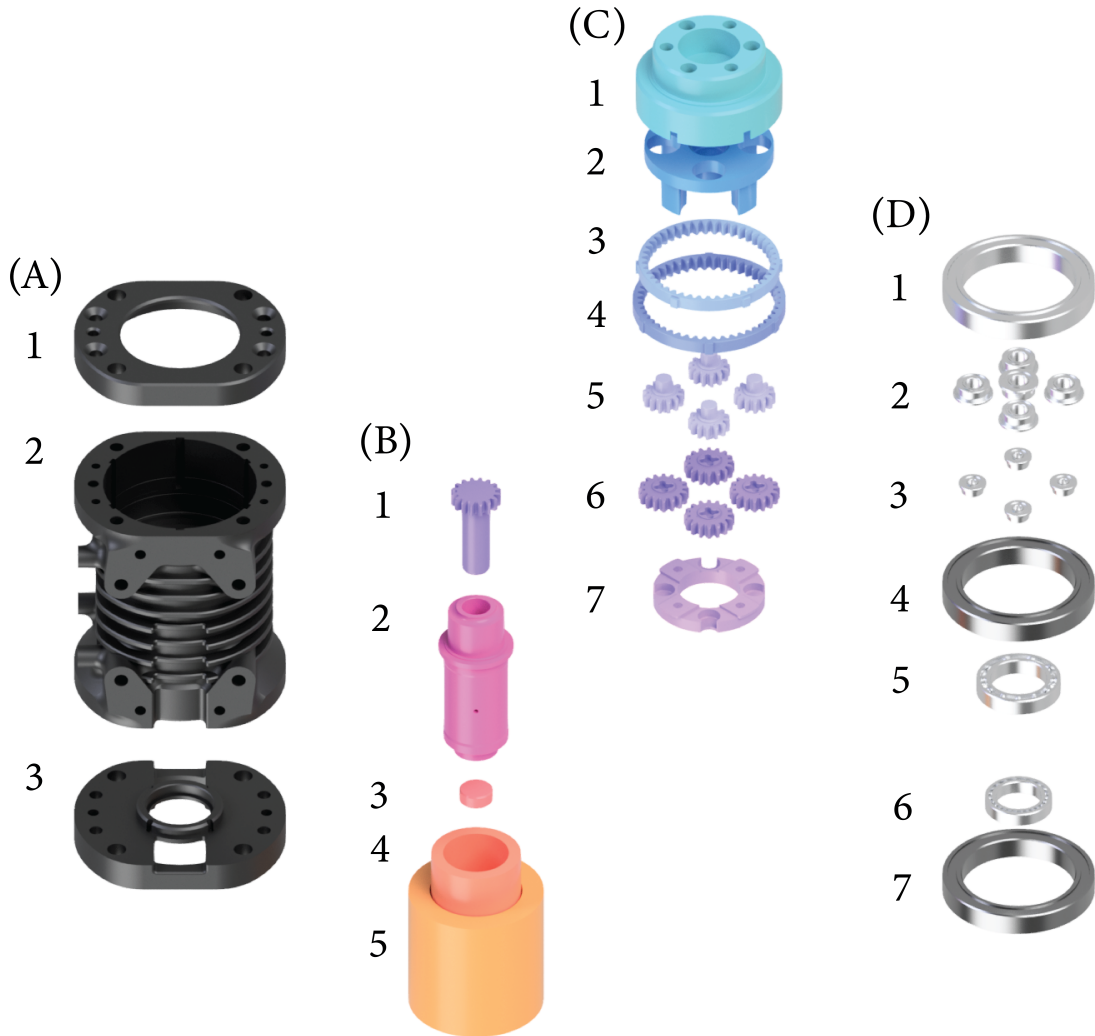


Figure 2.5: Exploded view of motor module, with components organized into subassemblies (A) housing, (B) motor, (C) gearbox, and (D) bearings.

Cutouts in the housing bottom (B3) visible in Fig. 2.5 allow for wires internal to the module (both from the motor itself and from its encoder board) to be passed through the housing, routed between the heat dissipation fins, and connected to the driver board at its mounting location. Power and communication cables routed between modules also pass through these cutouts, and their management is discussed further in Chapter 3.

I.D. Number (Fig. 2.5)	Component Name
A1	Housing Top
A2	Housing Body
A3	Housing Bottom
B1	Sun Gear with Shaft
B2	Rotary Shaft
B3	Encoder Magnet
B4	Mosrac U2523 Rotor
B5	Mosrac U2523 Stator
C1	Ring Gear Housing (Motor Output)
C2	Planetary Carrier Top
C3	Upper Stage Ring Gear
C4	Lower Stage Ring Gear
C5	4x Upper Stage Planetary Gears
C6	4x Lower Stage Planetary Gears
C7	Planetary Carrier Bottom
D1	McMaster-Carr No. 6656K198 Shielded Ball Bearing (1/3)
D2	5x McMaster-Carr No. 57155K537 Flanged Ball Bearings
D3	4x McMaster-Carr No. 57155K519 Flanged Ball Bearings
D4	McMaster-Carr No. 6656K198 Shielded Ball Bearing (2/3)
D5	McMaster-Carr No. 6656K69 Open Ball Bearing
D6	McMaster-Carr No. 7487N34 Open Ball Bearing
D7	McMaster-Carr No. 6656K198 Shielded Ball Bearing (3/3)

Table 2.2: All motor module components, excluding pins and fasteners.

Images of an assembled motor module after manufacturing are shown in Fig. 2.6. The aluminum housing has a black anodized finish, and internal components are a combination of machined aluminum and steel. The larger PCB mounted externally to the side of the housing is the driver board for the motor, handling both power and communication to the motor and from the encoder. The smaller PCB mounted underneath the bearing visible on the module in Fig. 2.6(b) is the encoder board, mounted directly above (according to the orientation in this image) the encoder magnet embedded in the rotary shaft.

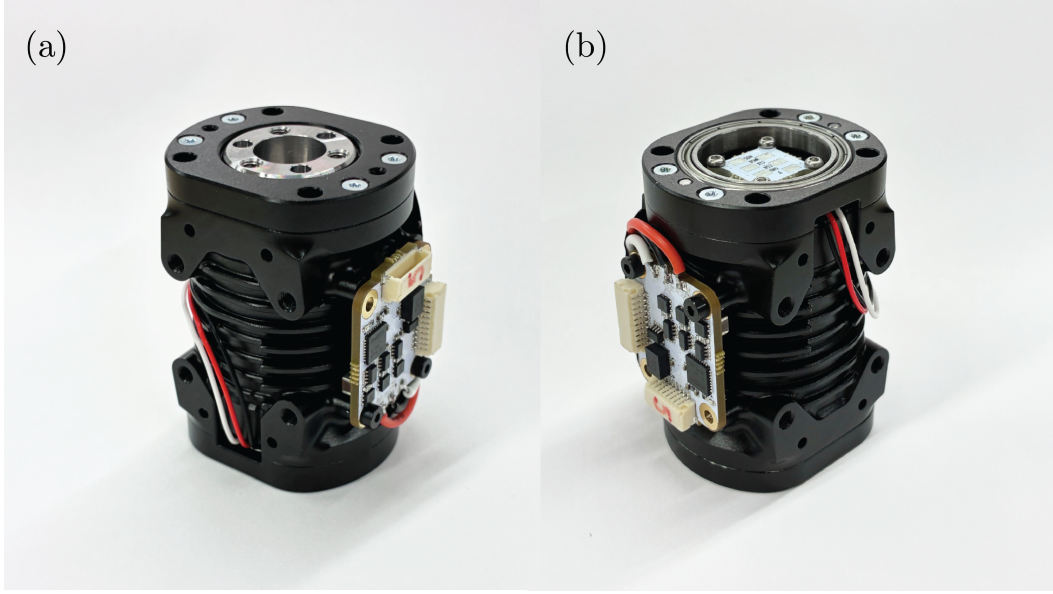


Figure 2.6: An example of an assembled motor module after manufacturing. (a) The module oriented with the gearbox output facing upwards, corresponding to the left cross section displayed above in Fig. 2.3. (b) The module oriented with the encoder board facing upwards, corresponding to the right cross section in Fig. 2.3.

2.4 Shrink fit feasibility analysis

The sun gear (Fig. 2.5, B1) and rotary shaft (Fig. 2.5, B2) in this motor module are designed with the option to be joined with a shrink fit, rather than bonding compound, to maximize concentricity and rotational stiffness in the module. Two versions of the rotary shaft, with interferences corresponding to a press fit and shrink fit, respectively, were manufactured for this motor module. The press fit version was included as a safety measure in the case of any unanticipated issues with the shrink fit, since this motor module is the first designed in our lab to include a shrink fit.

The behavior of this shrink fit under a range of conditions was analyzed to ensure its viability, including (1) the sun gear, rotary shaft, and rotor at room temperature after assembly, (2) the sun gear, rotary shaft, and rotor under approximately uniform heating by the motor during high power operation, and (3) the sun gear and rotary shaft under the maximum reasonable temperature difference ΔT imposed to enable assembly.

The shrink fit under each of these conditions was modeled under an elastic-perfectly plastic assumption (with any viscous effects neglected, and plastic effects irrelevant under the applied loads), using Lamé's equations for the loading of thick-walled cylinders. The motor module is represented in the cylindrical coordinate system shown in Fig. 2.7(a). Due to the presence of a gap between the rotor and stator, the primary components involved in the shrink fit are only the sun gear, the rotary shaft, and the rotor. Analysis is limited to two dimensions, $\hat{\theta}$ and $\hat{r}$, since axial constraint of all included elements is assumed to be minor. All components are modeled as cylinders (solid or thick-walled) to maintain reasonable

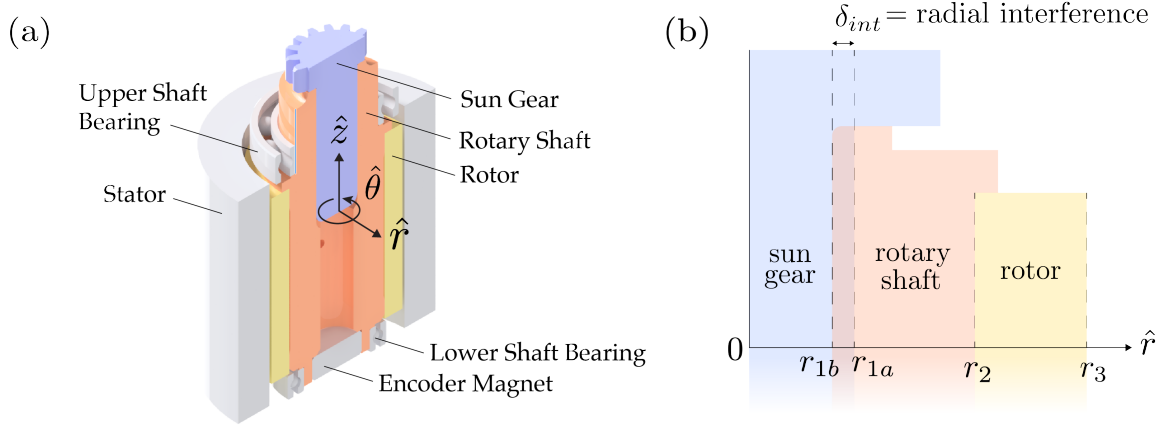


Figure 2.7: Shrink fit assembly overview. (a) Cross section of relevant assembly components; (b) Radial slice of shrink fit components, with important radii and interference indicated.

simplicity in the construction of the problem. The following geometric constraints can be assumed given the geometry shown in Fig. 2.7(b):

$$\delta_{int} = r_{1a} - r_{1b} \quad (2.1)$$

$$\delta_{int} = u_{1b} - u_{1a} \quad (2.2)$$

$$u_2^{rs} = u_2^{rot} \quad (2.3)$$

where δ_{int} is the radial interference between the sun gear and rotary shaft, r_{1a} is the undeformed radius of the sun gear, r_{1b} is the undeformed inner radius of the rotary shaft, and u_{1a} and u_{1b} are the radial displacement of the sun gear and rotary shaft, respectively, at their interface. u_2^{rs} and u_2^{rot} are the radial displacement of the rotary shaft and rotor, respectively, at r_2 .

Additionally, the following constitutive relationships hold for such geometries:

$$u_r = r\varepsilon_{\theta\theta}(r) \quad (2.4)$$

$$\varepsilon_{\theta\theta}(r) = \varepsilon_{\theta\theta}^{\text{elastic}} + \varepsilon_{\theta\theta}^{\text{thermal}} \quad (2.5)$$

$$\varepsilon_{\theta\theta}^{\text{thermal}} = \alpha\Delta T \quad (2.6)$$

$$\varepsilon_{\theta\theta}^{\text{elastic}} = \frac{1}{E} (\sigma_{\theta\theta} - \nu\sigma_{rr}) \quad (2.7)$$

$$\sigma_{\theta\theta} = \frac{E}{1 - \nu^2} (\varepsilon_{\theta\theta} + \nu\varepsilon_{rr}) \quad (2.8)$$

where ε refers to strain, σ refers to stress, u refers to displacement, α is the coefficient of thermal expansion for a given material, ΔT is the temperature difference applied to a material

relative to some starting reference, E is the Young's modulus of the material, and ν is the Poisson's ratio of the material.

These geometric constraints and constitutive equations (specifically Eq. 2.2 and Eq. 2.4, followed by Eq. 2.6 and Eq. 2.7) can be combined into the following form:

$$\delta_{int} = r_{1b}\varepsilon_{\theta\theta}(r_{1b}) - r_{1a}\varepsilon_{\theta\theta}(r_{1a}) \quad (2.9)$$

$$\begin{aligned} &= r_{1b} \left[\frac{1}{E_{rs}} (\sigma_{\theta\theta}(r_{1b}) - \nu\sigma_{rr}(r_{1b})) + \alpha_{rs}\Delta T_{rs} \right] \\ &\quad - r_{1a} \left[\frac{1}{E_{sun}} (\sigma_{\theta\theta}(r_{1a}) - \nu\sigma_{rr}(r_{1a})) + \alpha_{sun}\Delta T_{sun} \right] \end{aligned} \quad (2.10)$$

We can then consider the equilibrium constraints provided by the system:

$$\sigma_{rr}^{sun} = \sigma_{rr}^{sun}(r_{1a}) \quad (2.11)$$

$$\sigma_{rr}^{sun}(r_{1a}) = \sigma_{rr}^{rs}(r_{1b}) \quad (2.12)$$

$$\sigma_{rr}^{rs}(r_2) = \sigma_{rr}^{rot}(r_2) \quad (2.13)$$

$$\sigma_{rr}^{rot}(r_3) = 0 \quad (2.14)$$

where it can be noted that the stresses within the sun gear, which is a solid cylinder, are constant.

Lamé's equations [17] for a thick-walled cylinder under compressive internal and external pressures P_a and P_b , respectively, can be used in conjunction with equilibrium constraints to reveal stress as a function of radial position in the assembly of cylinders.

$$\sigma_{rr}(r) = \frac{P_a a^2}{b^2 - a^2} \left(1 - \frac{b^2}{r^2} \right) - \frac{P_b b^2}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right) \quad (2.15)$$

$$\sigma_{\theta\theta}(r) = \frac{P_a a^2}{b^2 - a^2} \left(1 + \frac{b^2}{r^2} \right) - \frac{P_b b^2}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right) \quad (2.16)$$

In this case, drawing from our equilibrium equations Eq. 2.12 and Eq. 2.13, P_a is equal to $\sigma_{rr}^{rs}(r_{1b})$, the interface pressure at the shrink fit, and P_b is equal to $\sigma_{rr}^{rs}(r_2)$, the interface pressure between the rotary shaft and rotor. This yields the following final equations, which present a fully defined system when substituted into Eq. 2.10 that can be used to solve for interface pressures, and subsequently stress distributions within each component.

$$\sigma_{rr}(r) = \frac{\sigma_{rr}^{rs}(r_{1b})r_{1b}^2}{r_2^2 - r_{1b}^2} \left(1 - \frac{r_2^2}{r^2} \right) - \frac{\sigma_{rr}^{rs}(r_2)r_2^2}{r_2^2 - r_{1b}^2} \left(1 - \frac{r_{1b}^2}{r^2} \right) \quad (2.17)$$

$$\sigma_{\theta\theta}(r) = \frac{\sigma_{rr}^{rs}(r_{1b})r_{1b}^2}{r_2^2 - r_{1b}^2} \left(1 + \frac{r_2^2}{r^2} \right) - \frac{\sigma_{rr}^{rs}(r_2)r_2^2}{r_2^2 - r_{1b}^2} \left(1 + \frac{r_{1b}^2}{r^2} \right) \quad (2.18)$$

Component	Sun Gear	Rotary Shaft	Rotor
Material	NAK80 Steel	7075-T6 Aluminum	1018 Steel
Young's Modulus (E)	206.8 GPa	71.7 GPa	205 GPa
Poisson's Ratio (ν)	0.29	0.33	0.29
Thermal Expansion Coefficient (α)	$12.6 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$	$23.2 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$	$12.6 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$
Yield Strength (σ_y)	1018 Mpa	503 MPa	370 Mpa

Table 2.3: Material properties relevant to the shrink fit.

The resulting stress distributions were solved for using SymPy, a symbolic math library in Python, using the following material properties:

Additionally, the geometric properties described below were used. Note that δ_{int}^{max} and δ_{int}^{min} refer to the maximum and minimum allowable values for the general shrink fit interference variable δ_{int} , obtained from Japanese Industrial Standard (JIS) B-0401, based on ISO Standard 286.

Symbolic variable	Numeric value
δ_{int}^{min}	4 μm
δ_{int}^{max}	11.5 μm
r_{1a}	2.5 mm
r_{1b}^{max}	2.496 mm
r_{1b}^{min}	2.4885 mm
r_2	5.8 mm
r_3	7.95 mm

Table 2.4: Geometric properties relevant to the shrink fit.

The resulting stress distributions in the case of both the minimum and maximum permissible interferences δ_{int} are plotted in Fig. 2.8. It can be seen that the radial stress profile exhibits the continuous behavior imposed by our boundary conditions, while hoop and von Mises stress profiles are free to experience discontinuities at material boundaries. Even in this maximal interference case, maximum von Mises stress in all materials does not exceed their respective yield strengths, listed in Table 2.4.

In order to validate the simplifying assumptions made during the analytical approach to solving for this stress distribution, the system was modeled in relevant loading cases (including those with thermal loads) in SolidWorks Simulation. Although some damping is present in the simulated result relative to the analytical solution, possibly as a result of a non-negligible mesh element size, the same functional form can be observed in both results.

In addition to the stress profiles at room temperature, two relevant cases of temperature change were considered: firstly, both the sun gear and rotary shaft under approximately uniform heating by the motor during high power operation, and secondly, the sun gear and rotary shaft under an opposing temperature difference ΔT imposed to enable assembly.

The case of uniform heating addresses the concern of problematic loosening due to non-

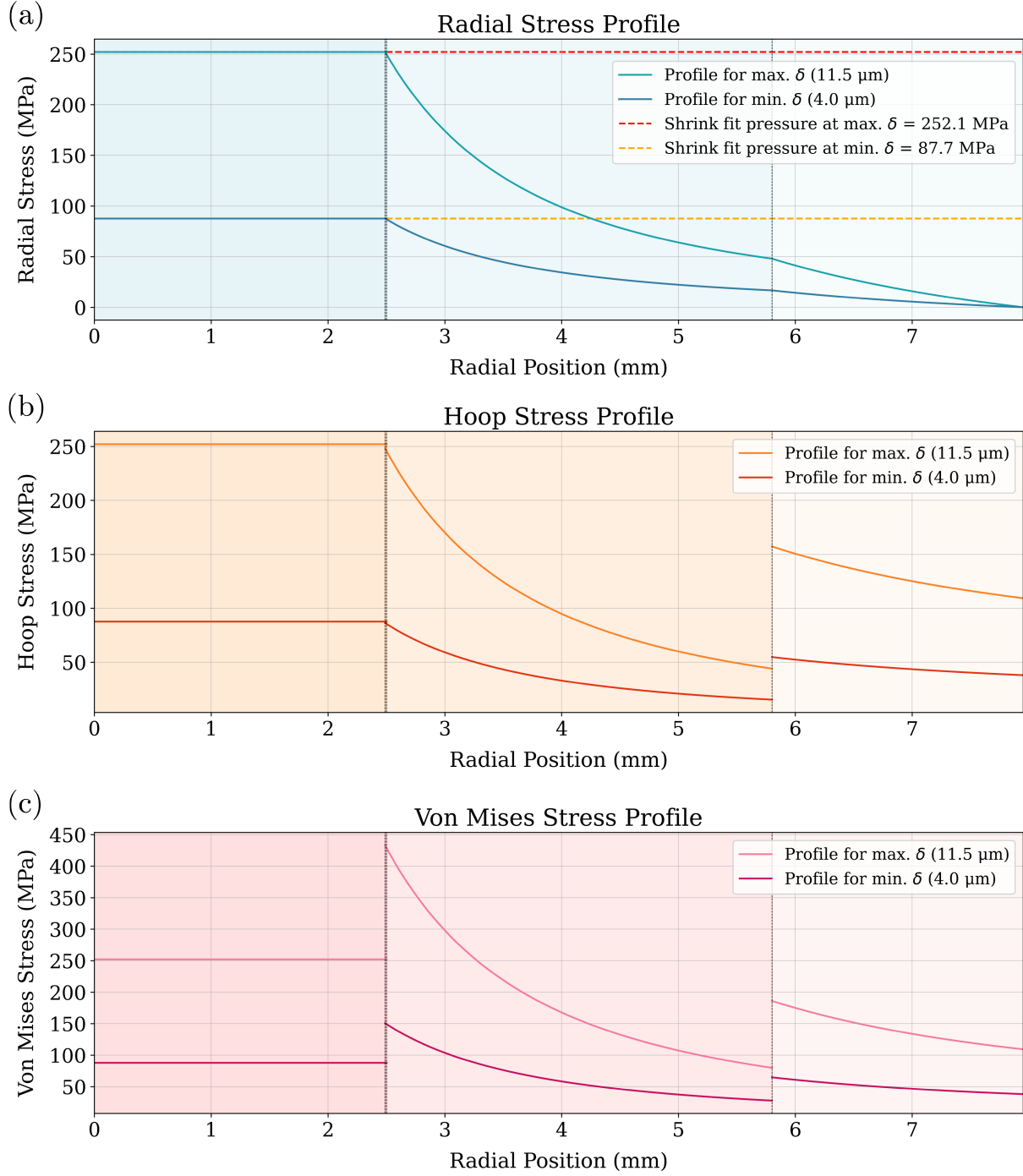


Figure 2.8: Analytical solutions for stress profiles in shrink fit assembly components. The system is considered under both extreme tolerance bounds of the shrink fit, δ_{int}^{min} and δ_{int}^{max} .

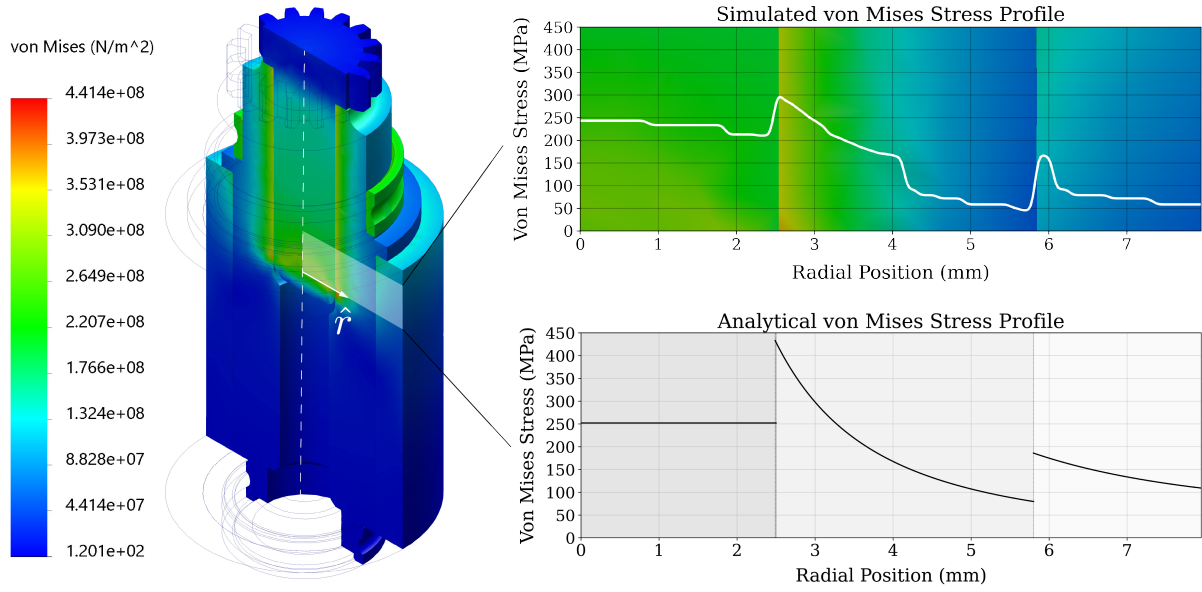


Figure 2.9: Comparison of simulation and analytical shrink fit stress profiles. The white curve superimposed on top of the simulated stress field shows an average of stress values at all visible heights at each radius. Note that the axial variation in the simulated stress field is a reflection of edge effects in the vicinity of the bottom of the sun gear, which are not modeled in the analytical solution.

uniform thermal expansion during operation. This case can be simply analyzed by imposing some nonzero ΔT to Eq. 2.10 introduced above. In this case, the ΔT applied to both components is one uniform value:

$$\begin{aligned}
 0 = r_{1b} \left[\frac{1}{E_{rs}} (\sigma_{\theta\theta}(r_{1b}) - \nu\sigma_{rr}(r_{1b})) + \alpha_{rs}\Delta T_{uniform} \right] \\
 - r_{1a} \left[\frac{1}{E_{sun}} (\sigma_{\theta\theta}(r_{1a}) - \nu\sigma_{rr}(r_{1a})) + \alpha_{sun}\Delta T_{uniform} \right]
 \end{aligned} \tag{2.19}$$

Additionally, the case of an opposing applied temperature difference addresses the feasibility of assembly, which in the case of a shrink fit involves the insertion of a thermally shrunk shaft (the sun gear) into a thermally expanded hole (the rotary shaft). Assembly by hand can be reasonably considered achievable when the interference at the interface of the parts in question has reached a clearance fit, meaning $\delta_{int} = 0$. By imposing this alternative boundary condition on Eq. 2.10, shown below:

$$\begin{aligned}
 0 = r_{1b} \left[\frac{1}{E_{rs}} (\sigma_{\theta\theta}(r_{1b}) - \nu\sigma_{rr}(r_{1b})) + \alpha_{rs}\Delta T_{rs} \right] \\
 - r_{1a} \left[\frac{1}{E_{sun}} (\sigma_{\theta\theta}(r_{1a}) - \nu\sigma_{rr}(r_{1a})) + \alpha_{sun}\Delta T_{sun} \right]
 \end{aligned} \tag{2.20}$$

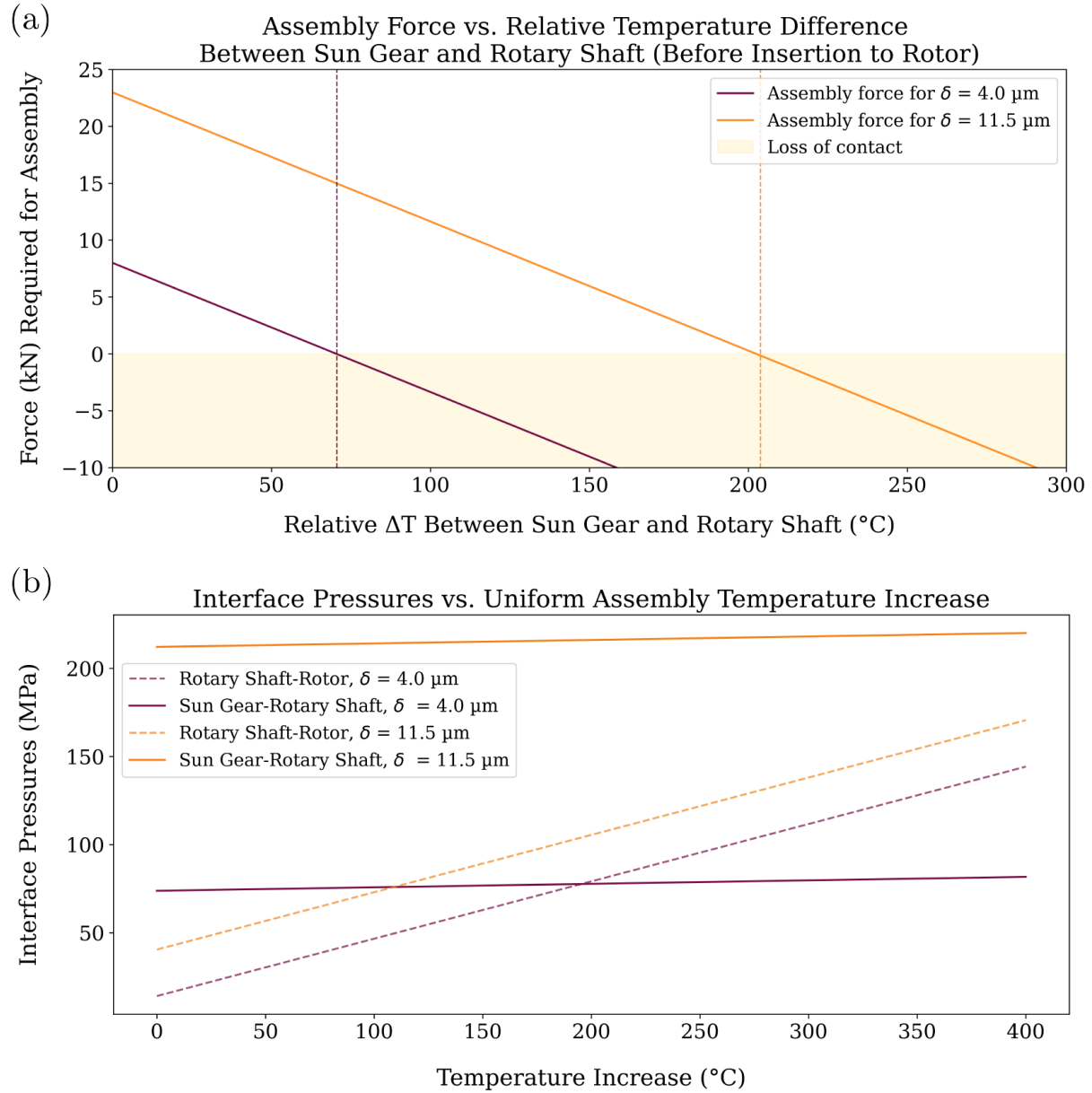


Figure 2.10: Thermal effects on shrink fit. (a) Assembly force required for shrink fit as a function of temperature difference between components. Note that loss of contact (LOC) occurs at 70.6°C for an interference of $4 \mu\text{m}$, and 203.8°C for an interference of $11.5 \mu\text{m}$. (b) It is not an immediately intuitive conclusion that interface pressure at the shrink fit radius increases under uniform temperature increase, given the steel sun gear's smaller coefficient of thermal expansion, α , relative to the aluminum rotary shaft. This phenomenon is exclusively a result of the rotary shaft being constrained at its outer radius by the steel rotor.

This boundary condition can be achieved when the difference in ΔT between the sun gear and rotary shaft reaches a critical value, with ΔT_{sun} becoming increasingly negative relative to ΔT_{rs} . These values are indicated in Fig. 2.10.

2.5 Summary

In this chapter, the design of a custom motor module for use in a torque dense, modular robotic wrist assembly was overviewed. Existing work including a compound planetary gearbox design incorporated by this module was described, and preliminary simulations to determine an appropriate motor for the module were summarized. Components of the module and their interfaces were outlined, and both analytical and simulation results regarding a shrink fit between two components were discussed.

Chapter 3

Wrist Assembly Design

With the integration of the custom motor modules described in the previous chapter, a compact, torque-dense 3 DOF wrist assembly was designed. A serial configuration of joints, rather than a parallel or spherical one, is used for easier modularity, simpler kinematics, and better future compatibility with reinforcement learning as a potential technique for control. The constraints and design priorities governing components of the assembly are discussed below, as well as a solution for fully internal wire routing.

3.1 Wrist assembly overview

An exploded view of each degree of freedom in the wrist is shown in Fig. 3.1.

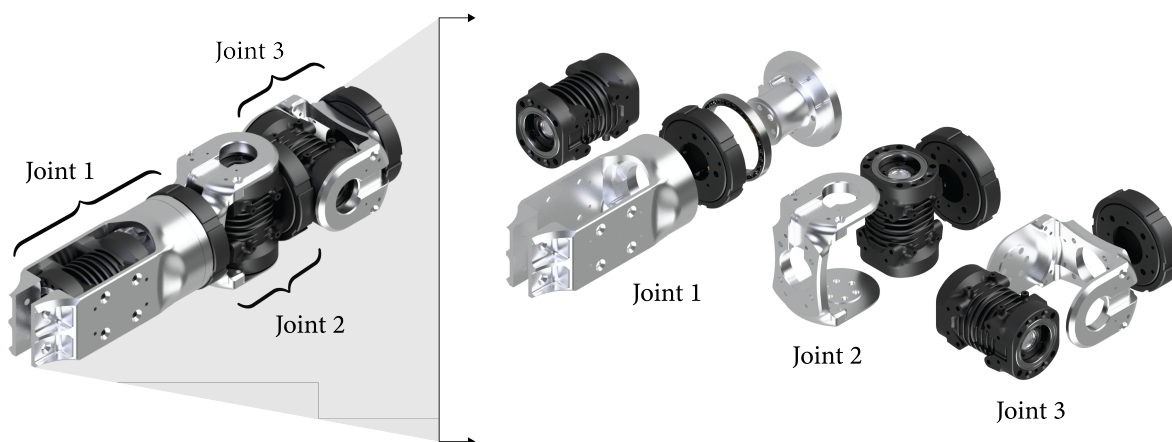


Figure 3.1: Exploded view of the wrist assembly, with components visually grouped by the degree of freedom that they govern.

Throughout the assembly, widely compatible mounting and integration components are designed to be patterned multiple times to bring down manufacturing cost and complexity. As a result, this assembly includes the small set of distinct components listed in Table 3.1.

The interchangeable output of the wrist (not pictured in Fig. 3.1 or Fig. 3.2) is connected through a torque transfer coupling, allowing for the rapid swapping of different end effectors

Number in Fig. 3.2	Component	No. Component Instances	No. Comprised Machined Parts
1	Internal Roll Shaft	1	1
2	Roll Support	1	3
3	Torque Transfer Coupling	3	3
4	Motor Mounting Bracket	2	3

Table 3.1: Names and instances of wrist assembly components.

during wrist use. Each degree of freedom in the assembly is separated by such a coupling, allowing users to easily add and remove joints to and from the wrist, even during active use of the system. For example, the same end effector can be mounted to component 3_A, 3_B, or 3_C in Fig. 2.3, resulting in a wrist with 1, 2, and 3 degrees of freedom, respectively.

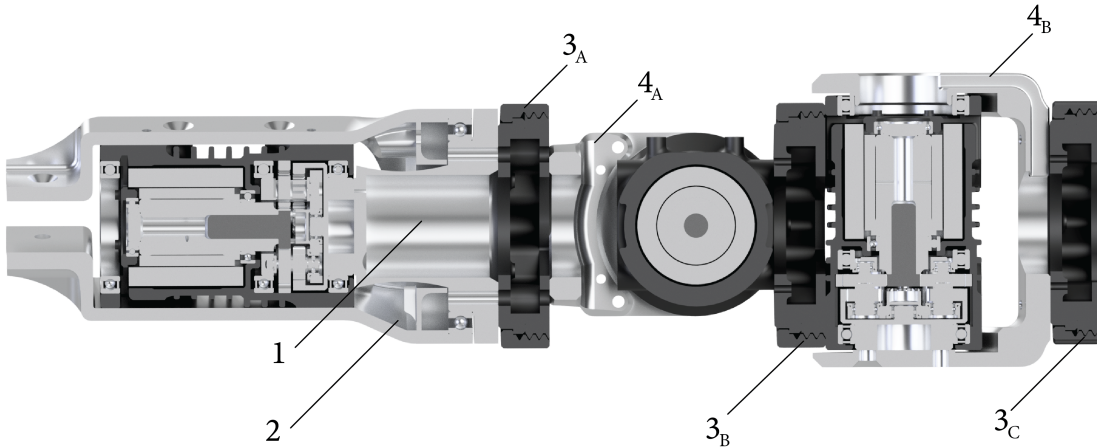


Figure 3.2: Cross section of the wrist assembly, exposing the internal structure of several components.

3.2 Torque transfer coupling

To enable easy assembly and disassembly of the wrist, both for rapid modification within a system and transfer between multiple systems in the lab, a low profile externally threaded coupling was designed. Although it is referred to here simply as a torque transfer coupling for brevity, it serves to safely and efficiently transmit both torques and forces between joints, with the substantial external thread handling compressive and tensile loads, and the large contact area of a Torx-style torque transfer profile within the coupling handling rotational loads. By unscrewing the threaded axial compression ring shown in Fig. 3.3, any two components that have been mounted to the upper and lower torque transfer disks can be separated from one another with ease. A large center hole in these disks allows wires to pass through the

coupling, and a simple wire pass-through PCB allows for the clean termination of power and communication (CAN, in this system) wires between joints of the wrist.

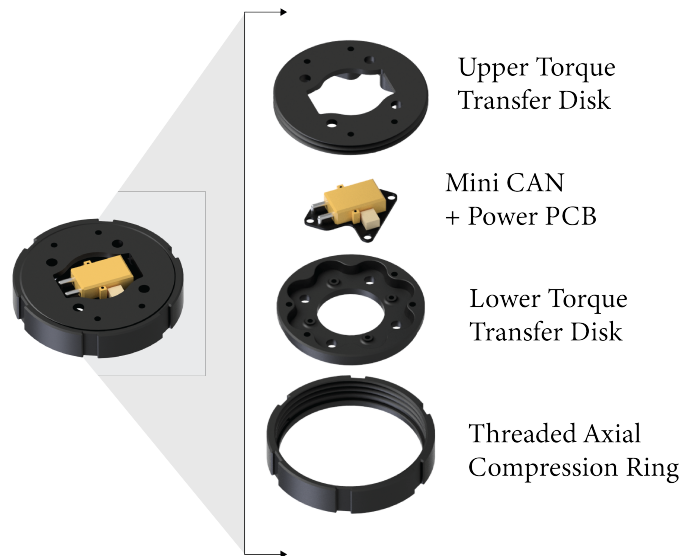


Figure 3.3: Exploded view of an externally threaded torque transfer coupling with an integrated wire pass-through PCB.

3.3 Wire routing

External wiring has been an inconvenience of some existing robotic arms in the lab, including the previous version of the arm utilized in the robotic table tennis system. Wire harnesses are difficult to make flexible and compact, and even well organized externally routed wires are at risk for tugging and crushing during high speed trajectories without significant protective measures. To minimize these negative effects, one goal of this wrist design was to feature fully internal wire routing. An overview of the unique internal wire management scheme designed for this assembly is shown in Fig. 3.4. Note that this figure shows only the wire path through the first and second joints in the wrist, since the passage of wires into the third joint is trivial in this design.

Complete range of motion, limited only by hard stops in the joints themselves, is maintained in the second and third joints by the passage of wires through channels in the motor mount brackets, through a hollow shaft in a bearing in the motor module itself, through a cutout between the bearing and rotor in the housing bottom and housing (protected by the housing bottom), and along paths between the fins on the housing. This cutout can be seen in Fig. 2.5.

The focus of the description of wire management in this section, however, lies on the first joint (wrist roll). This joint is an especially challenging case for internal wire routing, as a solution must be found for wires from the exterior of the roll motor to pass through the rotating output shaft of this motor and interface with the second motor. Several experimental

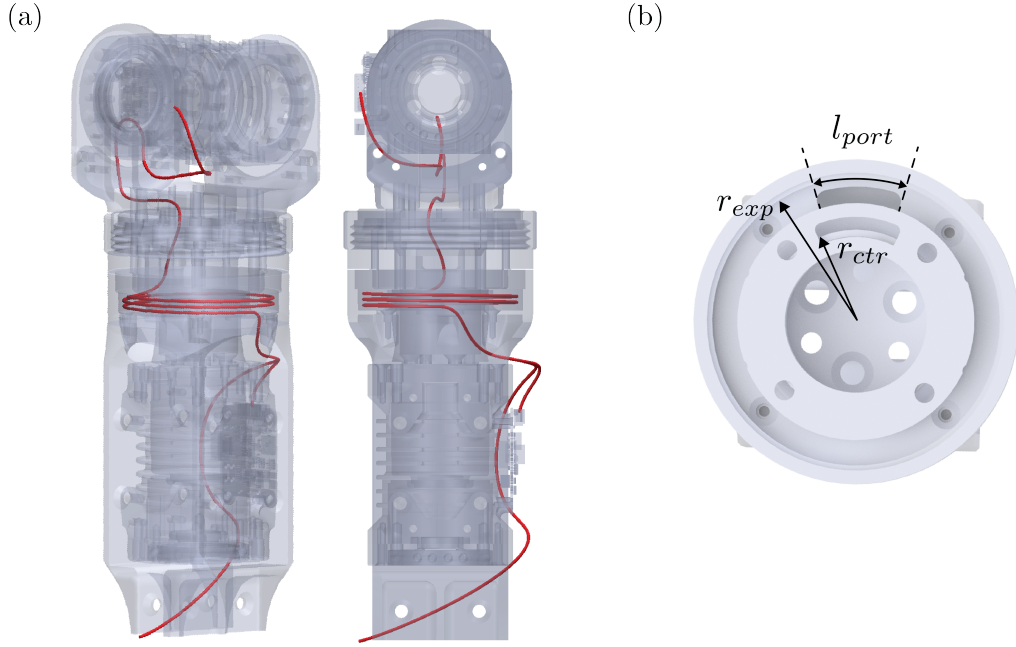


Figure 3.4: Wire routing setup. (a) The path of the wires through wrist joints 1 and 2, featuring a compact wire storage helix for roll range of motion. (b) Terminology used in calculations describing helical expansion wire management.

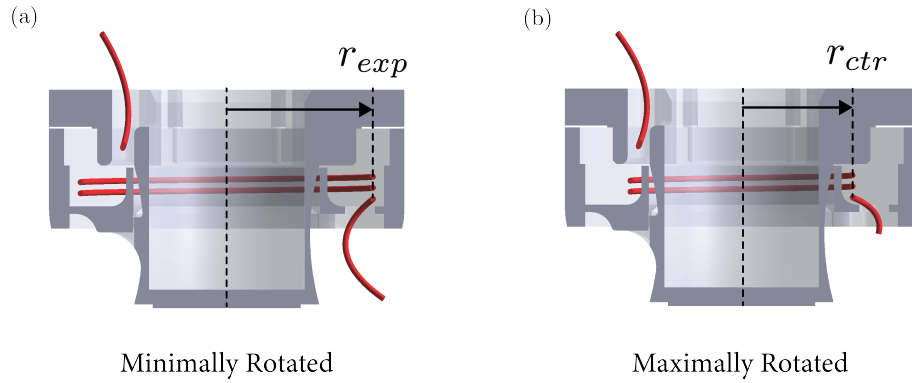


Figure 3.5: Helical expansion wire routing solution. (a) With the wrist roll joint in its minimally rotated position (as far as possible from the hard stop), the wire storage helix is fully expanded. (b) With the wrist roll joint in its maximally rotated position (just before contact with the hard stop), the wire storage helix is fully contracted.

concepts were developed during a broad ideation phase, and the most effective of these concepts is described in further detail below.

This geometrically simple, low-risk solution takes advantage of the smooth collapsibility of length present in a helical stretch of wiring (the wire storage helix). The required wire length to store can be conservatively described as the circumference of a fully expanded helix of wire, minus the length of the entry and exit ports on the top and bottom of the cavity storing the helix. It may be noted that travel limitation to a value slightly under 360 degrees by the hard stop is ignored in these simple calculations, causing them to be marginally more of an overestimate. This length can be written as ΔL_{req} as follows:

$$\Delta L_{req} = 2\pi r_{exp} - l_{port} \quad (3.1)$$

Additionally, the available expansion in length between two helical configurations, neglecting axial changes in length (that is, assuming a fairly flat helix in which $r_{helix} \gg h_{helix}$), takes the following form:

$$\Delta L_{helix} = N_{coils} \cdot 2\pi(r_{exp} - r_{ctr}) \quad (3.2)$$

Where r_{exp} is the fully expanded radius of the helix, and r_{ctr} is the fully contracted radius of the helix.

The constraint on minimum feasible difference in radius between the two helical configurations can then be found by equating Eq. 3.1 and Eq. 3.2. This constraint is solved below assuming 3 full coils and a port length of $\frac{1}{6}$ of the circumference, as shown in Fig. 3.4(b).

$$\frac{10\pi r_{exp}}{6} = N_{coils} 2\pi(r_{exp} - r_{ctr}) \quad (3.3)$$

$$r_{exp}^{min} \approx 1.39r_{ctr} \quad (3.4)$$

This minimum ratio served as a constraint to inform the geometry of the roll support and internal roll shaft components, which currently feature a ratio $\frac{r_{exp}}{r_{ctr}}$ of 1.56.

3.4 Summary

In this chapter, the design of a modular wrist assembly using the custom motor modules described in Chapter 2 was overviewed. Joint ordering was described, noting the use of identical parts for multiple roles in the wrist as a means to decrease manufacturing cost. The design of an externally threaded torque transfer coupling with an integrated power and communication pass-through PCB utilized repeatedly in the assembly was shown. Lastly, a custom helical expansion-based wire management technique to enable complete internal wire routing the wrist was introduced.

Chapter 4

Simulation & Trajectory Optimization

One exciting application of this wrist module is the exploration of the roles played by the wrist in arm trajectories. A series of simulation experiments was conducted with the wrist, analyzing how multiple factors in its use affect the performance of the arm.

Firstly, the degree of passivity of the wrist, modeled by a variable torque limit imposed on the wrist motors (with 0 allowable torque output representing an entirely passive wrist, and unchanged torque output representing an entirely active wrist).

Secondly, the extent of wrist utilization in the trajectory, modeled by a variable weight multiplier on position, velocity, and acceleration costs for the wrist joints independent of the rest of the arm. An extremely large maximum weight multiplier results in complete kinematic exclusion of the wrist from the trajectory (wrist "locking"), a multiplier of one representing typical use, and a minimum multiplier close to zero resulting in amplified wrist use relative to other joints in the arm.

4.1 Setup & workflow

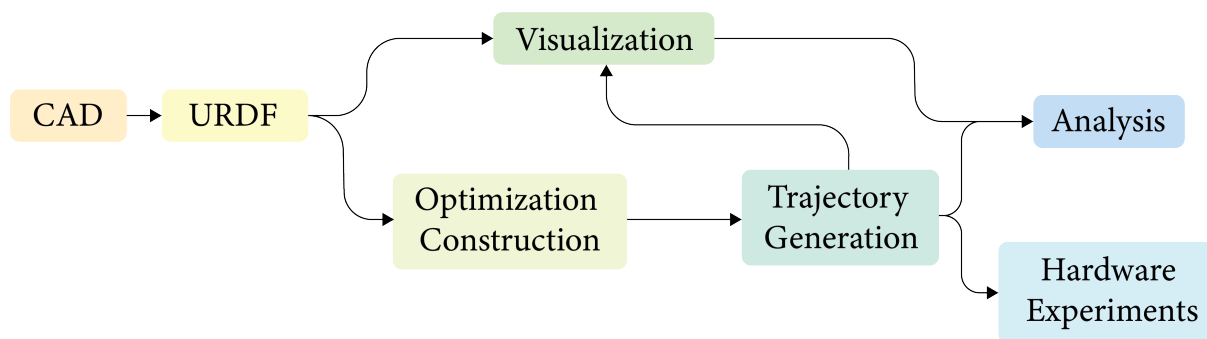


Figure 4.1: Flowchart of trajectory optimization workflow for both current simulation experiments and future hardware experiments.

The setup for these optimizations begins with the construction of a URDF from a minimal CAD assembly of the arm. URDFs were exported directly from SolidWorks using the SolidWorks to URDF Exporter add-in developed and maintained by Dr. Stephen Brawner.

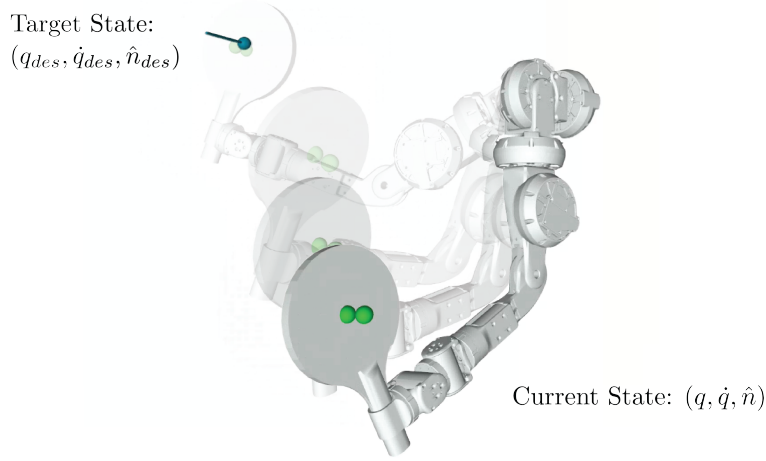


Figure 4.2: Overview of the arm model as rendered in MuJoCo. Shown as a small teal sphere in the upper left of the figure is the target position, with the line segment representing the direction of the target paddle normal vector (governing the paddle orientation during a strike). Shown in green on the paddle are two spheres, one representing the true paddle position, and the other serving as a visual indicator of paddle orientation.

CasADi’s nonlinear optimization tools were used to construct the optimizations, which lie under the following general structure:

$$\min_x f(x) \quad (4.1)$$

$$\text{s.t. } g_i(x) \leq 0 \quad \forall i \in \{1, \dots, m\} \quad (4.2)$$

$$h_j(x) = 0 \quad \forall j \in \{1, \dots, p\} \quad (4.3)$$

where x represents the decision variables to be optimized, $f(x)$ is the cost function, and $g(x)$ and $h(x)$ are inequality and equality constraints, respectively, that must be obeyed by the decision variables.

In the case of this optimization, the decision variables and cost function are the following:

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} q_0 \\ \vdots \\ q_6 \end{bmatrix} \quad (4.4)$$

$$f(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = C_p \|\mathbf{q} - \mathbf{q}_0\|^2 + C_v \|\dot{\mathbf{q}}\|^2 + C_a \|\ddot{\mathbf{q}}\|^2 \quad (4.5)$$

where $\mathbf{q}$, $\dot{\mathbf{q}}$, and $\ddot{\mathbf{q}}$ are the angular position, velocity, and acceleration, respectively, of all 7 joints in the arm. C_p , C_v , and C_a are scalar weights on the cost of each decision variable.

Since the application of robotic table tennis requires a very accurate match of the target state at strike, this target state is included as a constraint rather than a part of the cost

function. Additionally, the system is subject to dynamics constraints and state constraints at the start and end of the trajectory. Each trajectory is discretized into N nodes, starting at some initial state $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ at node 0, reaching the target state $(\mathbf{q}_{strike}, \dot{\mathbf{q}}_{strike})$ at some intermediate node n_{strike} , and ultimately returning to the initial state at the final node of the trajectory, n_f . The cost function across all of these nodes is minimized.

$$\min_{\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}} \sum_{n=0}^{N-1} C_p \|\mathbf{q}[n] - \mathbf{q}_0\|^2 + C_v \|\dot{\mathbf{q}}[n]\|^2 + C_a \|\ddot{\mathbf{q}}[n]\|^2 \quad (4.6)$$

$$\mathbf{q}[0] = \mathbf{q}_0 \quad (4.7)$$

$$\dot{\mathbf{q}}[0] = \dot{\mathbf{q}}_0 \quad (4.8)$$

$$\mathbf{q}_{min} \leq \mathbf{q}[n] \leq \mathbf{q}_{max} \quad \forall n \quad (4.9)$$

$$\dot{\mathbf{q}}_{min} \leq \dot{\mathbf{q}}[n] \leq \dot{\mathbf{q}}_{max} \quad \forall n \quad (4.10)$$

$$\tau_{min} \leq ID(\mathbf{q}[n], \dot{\mathbf{q}}[n], \ddot{\mathbf{q}}[n]) \leq \tau_{max} \quad \forall n \quad (4.11)$$

$$\mathbf{q}[n+1] = \mathbf{q}[n] + \dot{\mathbf{q}}[n]\Delta t \quad \forall n \quad (4.12)$$

$$\dot{\mathbf{q}}[n+1] = \dot{\mathbf{q}}[n] + \ddot{\mathbf{q}}[n]\Delta t \quad \forall n \quad (4.13)$$

$$\|FK(\mathbf{q}_{strike}) - \mathbf{p}_{des}\|_2^2 = 0 \quad (4.14)$$

$$\|J(\mathbf{q}_{strike})\dot{\mathbf{q}}_{strike} - \mathbf{v}_{des}\|_2^2 = 0 \quad (4.15)$$

$$\|\hat{\mathbf{n}}(\mathbf{q}_{strike}) - \hat{\mathbf{n}}_{des}\|_2^2 = 0 \quad (4.16)$$

$$\mathbf{q}_f = \mathbf{q}_0 \quad (4.17)$$

$$\dot{\mathbf{q}}_f = \dot{\mathbf{q}}_0 \quad (4.18)$$

where ID is the inverse dynamics of a given state, FK is the forward kinematics of a given position, J is the Jacobian of the paddle, $\hat{\mathbf{n}}$ is the normal unit vector of the paddle, τ_{min} and τ_{max} are the torque limits of the motors, $\dot{\mathbf{q}}_{min}$ and $\dot{\mathbf{q}}_{max}$ are the velocity limits of the motors, and $\mathbf{q}_{min}$ and $\mathbf{q}_{max}$ are the joint travel limits of the system. $\mathbf{p}_{des}$, $\mathbf{v}_{des}$, and $\hat{\mathbf{n}}_{des}$ are the target position, velocity, and normal vector (orientation) of the paddle.

In Fig. 4.2 the arm URDF is shown at an intermediate state in an example trajectory, having already met the target state and proceeding back to the initial state.

4.2 Variable wrist utilization

The first experiment conducted with these trajectory optimizations served as a preliminary examination of the role of the wrist in arm trajectories. Performance was analyzed over a sweep of trajectories through a fixed target state, with varying extents of wrist exclusion, or

"locking", achieved by modulating wrist-specific joint weights to multiply their contribution to the cost function.

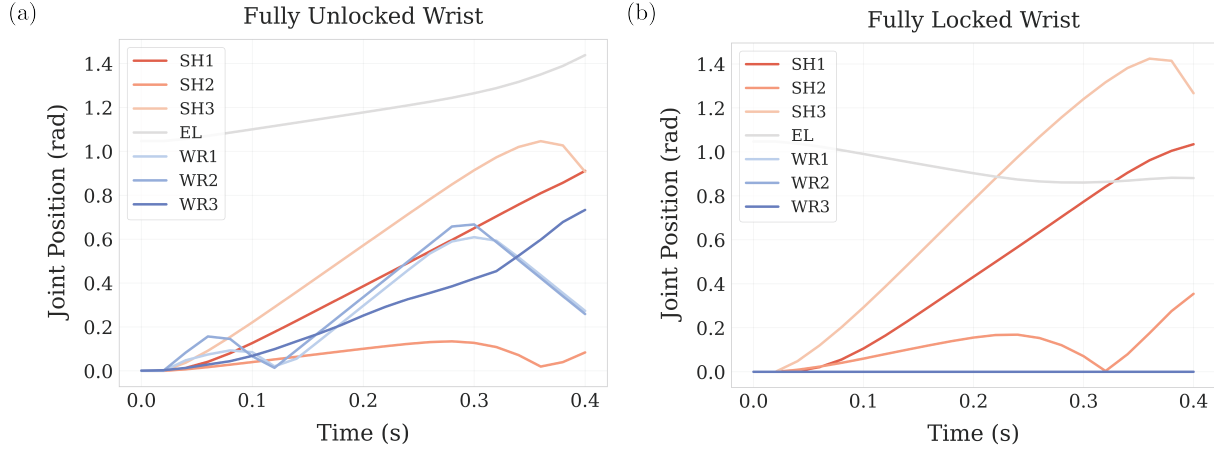


Figure 4.3: (a) Joint positions over the first portion of a trajectory for a fully unlocked (fully utilized) wrist, with strike time at 0.4 seconds. (b) Joint positions over the first portion of a trajectory with strike time at 0.4 seconds for a fully locked wrist.

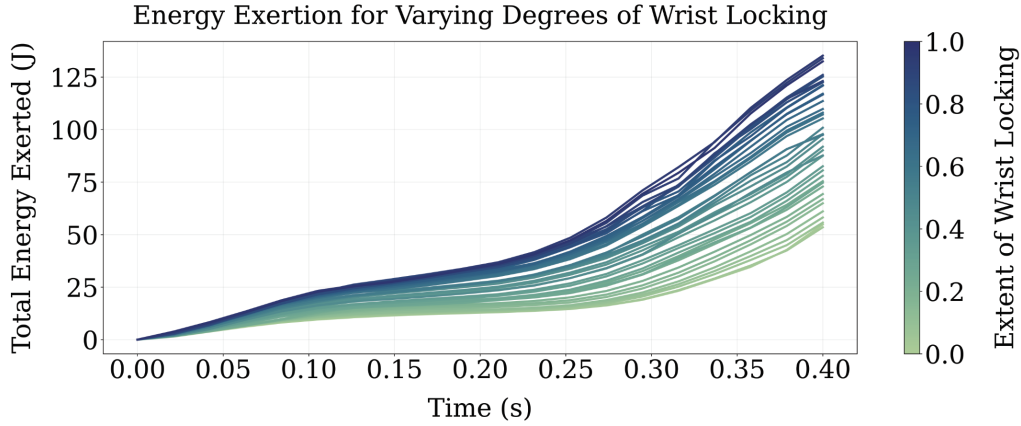


Figure 4.4: Energy exerted by all joints in the arm collectively over the first portion of a trajectory (from initial state to target state at 0.4 seconds), plotted for a sweep of varying degrees of wrist locking.

In Fig. 4.3 a comparison of joint positions during a trajectory in two extremes on this swept spectrum is shown. For a fully unlocked wrist, wrist costs are multiplied by approximately zero, allowing the wrist to be utilized even over the rest of the arm. For a fully locked wrist, wrist costs are multiplied by approximately 10^6 , causing them to be effectively kinematically excluded from the trajectory (acting as a single rigid body, rather than 3 DOFs). In reference to the optimization formulation described in the section above, this sweep introduced a scaling parameter s to the cost function from Eq. 4.6 in the previous section:

$$\min_{\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}} \sum_{n=0}^N \left(\mathbf{C}'_p \odot \|\mathbf{q}[n] - \mathbf{q}_0\|^2 + \mathbf{C}'_v \odot \|\dot{\mathbf{q}}[n]\|^2 + \mathbf{C}'_a \odot \|\ddot{\mathbf{q}}[n]\|^2 \right) \quad (4.19)$$

$$\mathbf{C}' = C\boldsymbol{\sigma}[\mathbf{j}]$$

$$\mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 6 \end{bmatrix} \quad (4.20)$$

$$\boldsymbol{\sigma}[i] = \begin{cases} 1, & 0 \leq i < 4 \\ s, & 4 \leq i \leq 6 \end{cases} \quad (4.21)$$

For this experiment, s was swept from 10^{-3} to 10^6 . Although the arm was able to reach a range of target positions with both the locked and unlocked wrist, increasing utilization of the wrist noticeably improved the efficiency of the arm over the course of the trajectory, with much lower energies being exerted by all joints in the arm collectively for trajectories with complete wrist utilization than those with wrist exclusion (Fig. 4.4).

4.3 Variable wrist passivity

The second experiment conducted with these trajectory optimizations explored the effect of the wrist as a passive element on the arm (capable of free motion, but no torque output) versus an active element (capable of the maximum torque output of each of the 3 motors in the wrist). This was achieved by the modulation of a joint-dependent multiplier a inserted to the torque limits of the arm from Eq. 4.11 in section 4.1:

$$\boldsymbol{\tau}'_{min} \leq ID(\mathbf{q}[n], \dot{\mathbf{q}}[n], \ddot{\mathbf{q}}[n]) \leq \boldsymbol{\tau}'_{max} \quad (4.22)$$

$$\boldsymbol{\tau}' = \boldsymbol{\tau} \odot \boldsymbol{\alpha}[\mathbf{j}]$$

$$\mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 6 \end{bmatrix} \quad (4.23)$$

$$\boldsymbol{\alpha}[i] = \begin{cases} 1, & 0 \leq i < 4 \\ a, & 4 \leq i \leq 6 \end{cases} \quad (4.24)$$

For this sweep, a was varied between 0 to 1, representing the range from complete passivity to complete activation. For both passive and active wrists, the target states were reached with minimal error. However, it was observed that for a more active wrist, the error in paddle state approaches zero more quickly as the strike time is approached (shown with position errors in Fig. 4.5).

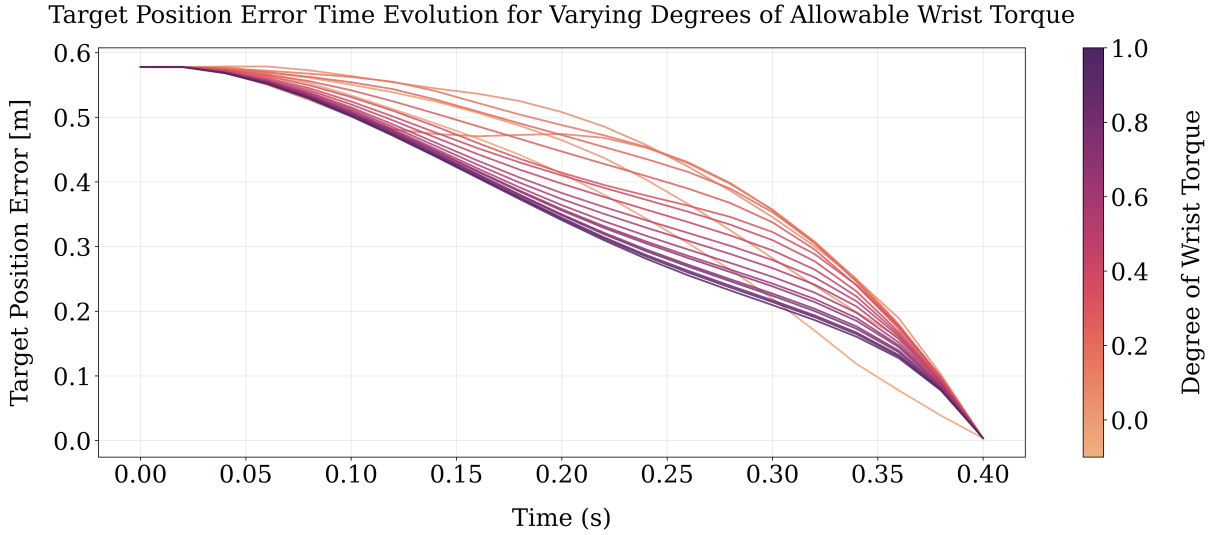


Figure 4.5: Position error of the paddle over the first portion of a trajectory (from initial state to target state at 0.4 seconds), plotted for a sweep of varying degrees of wrist activation (allowable torque).

Additionally, more active wrists were observed to enable more efficient trajectories, with the collective energy expenditure of all joints in the arm seeing a direct decrease in proportion to the degree of wrist activation. This is likely due to the greater degree of controllability offered by an active wrist; for a passive wrist, joints in the upper arm may need to reach more extreme torques and speeds in order to successfully maneuver the paddle into an acceptable state through the wrist without its assistance.

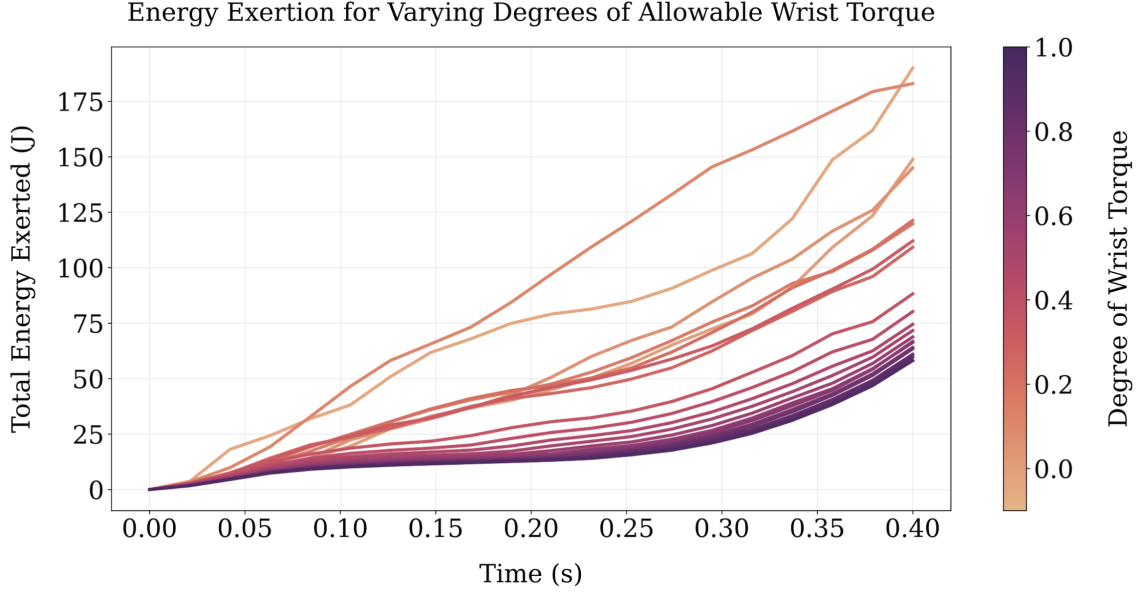


Figure 4.6: Energy exerted by all joints in the arm collectively over the first portion of a trajectory (from initial state to target state at 0.4 seconds), plotted for a sweep of varying degrees of wrist activity (allowable torque).

4.4 Summary

In this chapter, optimized trajectories were created and analyzed as a tool for understanding the role of the wrist in the dynamic performance of a robotic arm, specifically in the context of high speed robotic table tennis. Both the theoretical setup and practical implementation workflow for the optimizations were discussed. Two experimental sweeps of trajectories were run: firstly, a sweep analyzing the effect of the extent of wrist utilization, in which variable costs assigned to wrist joints set the degree of wrist utilization in a range of places between two extremes: full constraint of the wrist to behave as a single rigid body, or complete freedom for the wrist be used. Secondly, a sweep was conducted to analyze the effect of varying degrees of wrist passivity (in which wrist motors can output no torque) versus activity (in which the wrist motors can output full torque). Both sweeps revealed notable results regarding the energy expenditure of the arm throughout the trajectories, and the passivity sweep showed a clear trend of increasing wrist activity accelerating the rate of error reduction relative to the target state during trajectories.

Chapter 5

Conclusions

5.1 Review

The wrist is an integral contributor to human dexterity in both fine and gross manipulation tasks, and its complex anatomy makes it an especially interesting case to consider for bio-inspired systems. In this thesis, a custom 3 DOF wrist assembly and the motor modules that drive it were designed, and their improvements on the performance of a robotic arm were analyzed in simulation.

In order to achieve the necessary torque density to drive this system with the desired performance, a custom compact motor module with an inline gearbox was designed and manufactured. A wrist assembly built around these motors was discussed, with a particular focus on modularity. In this design, externally threaded torque transfer couplings allow for instant removal, addition, and reconfiguration of wrist joints, while a custom helical expansion-based wire management approach enables complete internal wire routing of the wrist.

Lastly, trajectory optimization was used as a tool for understanding the role of the wrist in the dynamic performance of a robotic arm, and testing the implementation of experimental control schemata (including passivity and variable wrist utilization) in simulation. Both sweeps revealed especially notable results regarding the energy expenditure of the arm throughout the trajectories, suggesting that use of an active wrist can allow for more efficient system operation. Additionally, accelerated target error reduction correlated with increased wrist activity, warranting further exploration of the wrist's utility not just as a contributor of last minute adjustments in kinetic chain-style trajectories, but as an immediately powerful tool from the onset of a trajectory.

5.2 Future work

There are extensive pathways for future work on this project, some of which are already in progress. The pursuit of both hardware experiments and more in-depth simulation experiments will provide concrete insights about the optimal hardware configuration and control techniques for wrist DOFs in dynamic robotic systems. Hardware characterization tests assessing the performance of the motor modules, including backlash quantification,

efficiency measurements, and torque-speed curve mapping will allow for their more realistic simulation. Hardware experiments with these new wrist DOFs integrated into the lab's robotic table tennis platform (currently equipped with only one degree of freedom at the wrist) will serve as a chance to push the system to its physical limits, validate and discuss hardware designs, and evaluate the sim-to-real gap present in the system when hardware performance is compared to simulated trajectories.

Additionally, higher dimensional trajectory optimization sweeps will reveal more information about the effects of wrist behavior observed in the preliminary sweeps conducted as part of this work. For example, a valuable follow-up experiment to the wrist utilization explorations discussed in section 4.2 would include two sweeps of a wide range of target positions, with a boolean "locking" setting on the wrist. Performance drop-off at increasingly difficult to reach positions (defined by both orientation and location in space) in these sweeps could be compared to estimate the effective workspace expansion provided by the wrist, especially in areas close to the base of the arm where self-collision is difficult to avoid.

Finally, the installation and testing of the wrist assembly across multiple platforms in the lab, including the MIT Humanoid, will allow the effectiveness of its modular design to truly be tested, while offering opportunities for a deeper dive into differences in the practical role of the wrist for fine, dexterous manipulation tasks versus high speed, dynamic tasks. This ultimately fits the long term vision of this project, allowing the design developed in this work for a torque-dense robotic wrist with flexible applications to expand the capabilities of as many distinct systems as possible.

References

- [1] Agility Robotics. *Digit: Human-Centric Bipedal Robot for Logistics and Physical Work*. <https://agilityrobotics.com/>. Accessed: 2026-04-28. 2024.
- [2] Shadow Robot Company. *Shadow Dexterous Hand Series: The World's Most Dexterous Humanoid Robot Hand*. <https://shadowrobot.com/dexterous-hand-series/>. Accessed: 2026-04-28. 2024.
- [3] *Design and Development of the MIT Humanoid: A Dynamic and Robust Research Platform* | IEEE Conference Publication | IEEE Xplore. URL: <https://ieeexplore.ieee.org/document/10375199> (visited on 04/28/2026).
- [4] *On Extracting Design Principles from Biology: I. Method-General Answers to High-Level Design Questions for Bioinspired Robots - IOPscience*. URL: <https://iopscience.iop.org/article/10.1088/1748-3190/10/1/016010> (visited on 04/28/2026).
- [5] B. Dynamics. *Atlas: Industrial Humanoid Robot Platform*. 2026. URL: <https://bostondynamics.com/products/atlas/> (visited on 04/28/2026).
- [6] E. B. Stern, S. R. Ytterberg, H. E. Krug, and M. L. Mahowald. "Finger Dexterity and Hand Function: Effect of Three Commercial Wrist Extensor Orthoses on Patients with Rheumatoid Arthritis." ISSN: 1529-0131. URL: [https://onlinelibrary.wiley.com/doi/10.1002/1529-0131\(199606\)9:3%3C197::AID-ANR1790090308%3E3.0.CO;2-W](https://onlinelibrary.wiley.com/doi/10.1002/1529-0131(199606)9:3%3C197::AID-ANR1790090308%3E3.0.CO;2-W) (visited on 05/03/2026).
- [7] J. R. Blackwell and K. J. Cole. "Wrist Kinematics Differ in Expert and Novice Tennis Players Performing the Backhand Stroke: Implications for Tennis Elbow." *Journal of Biomechanics*, **27**(5), May 1, 1994, pp. 509–516. ISSN: 0021-9290. DOI: [10.1016/0021-9290\(94\)90062-0](https://doi.org/10.1016/0021-9290(94)90062-0). URL: <https://www.sciencedirect.com/science/article/pii/S0021929094900620> (visited on 05/03/2026).
- [8] *The Kinetic Chain Revisited: New Concepts on Throwing Mechanics and Injury - Chu - 2016 - PM&R - Wiley Online Library*. URL: <https://onlinelibrary.wiley.com/doi/10.1016/j.pmrj.2015.11.015> (visited on 04/28/2026).
- [9] S. J. De Serres and T. E. Milner. "Wrist Muscle Activation Patterns and Stiffness Associated with Stable and Unstable Mechanical Loads." *Experimental Brain Research*, **86**(2), Sept. 1, 1991, pp. 451–458. ISSN: 1432-1106. DOI: [10.1007/BF00228972](https://doi.org/10.1007/BF00228972). URL: <https://doi.org/10.1007/BF00228972> (visited on 05/03/2026).
- [10] *High Speed Robotic Table Tennis Swinging Using Lightweight Hardware with Model Predictive Control* | IEEE Conference Publication | IEEE Xplore. URL: <https://ieeexplore.ieee.org/abstract/document/11127907> (visited on 04/28/2026).

- [11] A. Hattori. “Design of a high torque density modular actuator for dynamic robots.” Accessed: 2026-05-02. MA thesis. Massachusetts Institute of Technology, 2020. URL: <https://dspace.mit.edu/handle/1721.1/127165>.
- [12] *Synthesis of Multi-Degree of Freedom, Parallel Flexure System Concepts via Freedom and Constraint Topology (FACT) – Part I: Principles* - ScienceDirect. URL: <https://www.sciencedirect.com/science/article/pii/S0141635909000920> (visited on 04/29/2026).
- [13] E. B. Stanger-Jones. “Expanding the Capabilities of Dynamic Robotic Systems.” Accessed: 2026-05-02. MA thesis. Massachusetts Institute of Technology, 2022. URL: <https://dspace.mit.edu/handle/1721.1/143188>.
- [14] A. SaLoutos. “Development of a Hierarchical Reflexive Control Framework for Autonomous Robotic Manipulation.” Accessed: 2026-05-02. PhD thesis. Massachusetts Institute of Technology, 2025. URL: <https://dspace.mit.edu/handle/1721.1/163440>.
- [15] *Mini Cheetah: A Platform for Pushing the Limits of Dynamic Quadruped Control / IEEE Conference Publication / IEEE Xplore*. URL: <https://ieeexplore.ieee.org/document/8793865> (visited on 04/28/2026).
- [16] N. Cerone. *C-30: Design of a Compact Wolfrom Gearbox for High-Strength Dexterous Robotics*. 2026.
- [17] P. Jasion. *Thick-Walled Cylinders: Lamé Equation*. Poznań, 2024. URL: https://pawel.jasion.pracownik.put.poznan.pl/wp-content/uploads/2025/05/thick_walled_cylinder.pdf (visited on 09/16/2025).